7 Even More Graphing Rational Equations

Warmup - find the vertical asymptotes, if any.

1. $f(x) = \dfrac{x}{x+4}$ $x = -4$

2. $f(x) = \dfrac{x}{x-3}$ $x = 3$

3. $f(x) = \dfrac{x+3}{x(x+4)}$ $x = 0, -4$

4. $g(x) = \dfrac{x+3}{x(x-3)}$ $x = 0,\ 3$

5. $h(x) = \dfrac{x}{x(x+4)}$ $x = -4$

6. $f(x) = \dfrac{x}{x(x-3)}$ $x = 3$

Find the horizontal asymptotes, if any.

7. $f(x) = \dfrac{12x}{3x^2+1}$ $y = 0$

6. $f(x) = \dfrac{15x}{3x+1}$ $y = 5$

7 Even More Graphing Rational Equations

Asymptotes and Intercepts

Find the asymptotes and intercepts of the function.
Sketch the graph.

$$f(x) = \frac{2x^2 + 5x + 2}{x^2 - 9}$$

Vertical asymptotes

x = 3, x = -3

Horizontal asymptotes

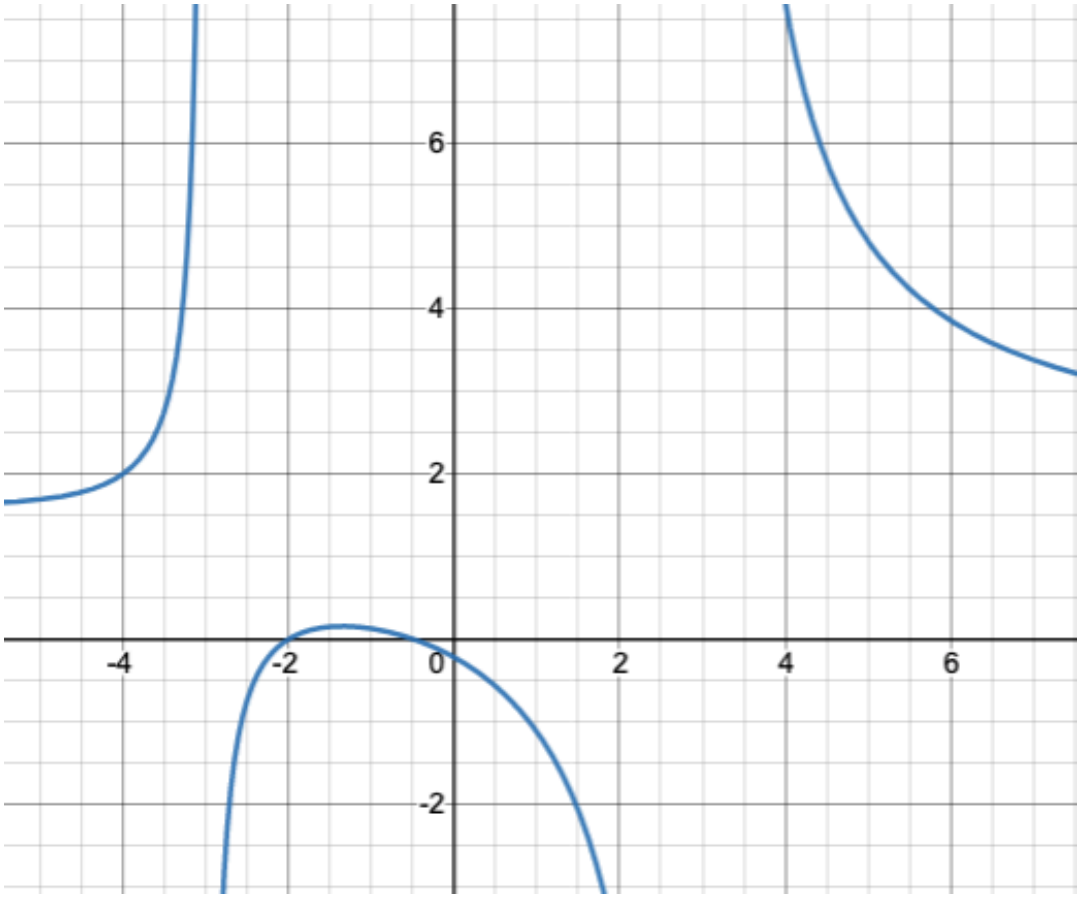
y = 2

x-intercepts

x = -2, x = $-\frac{1}{2}$

y-intercepts

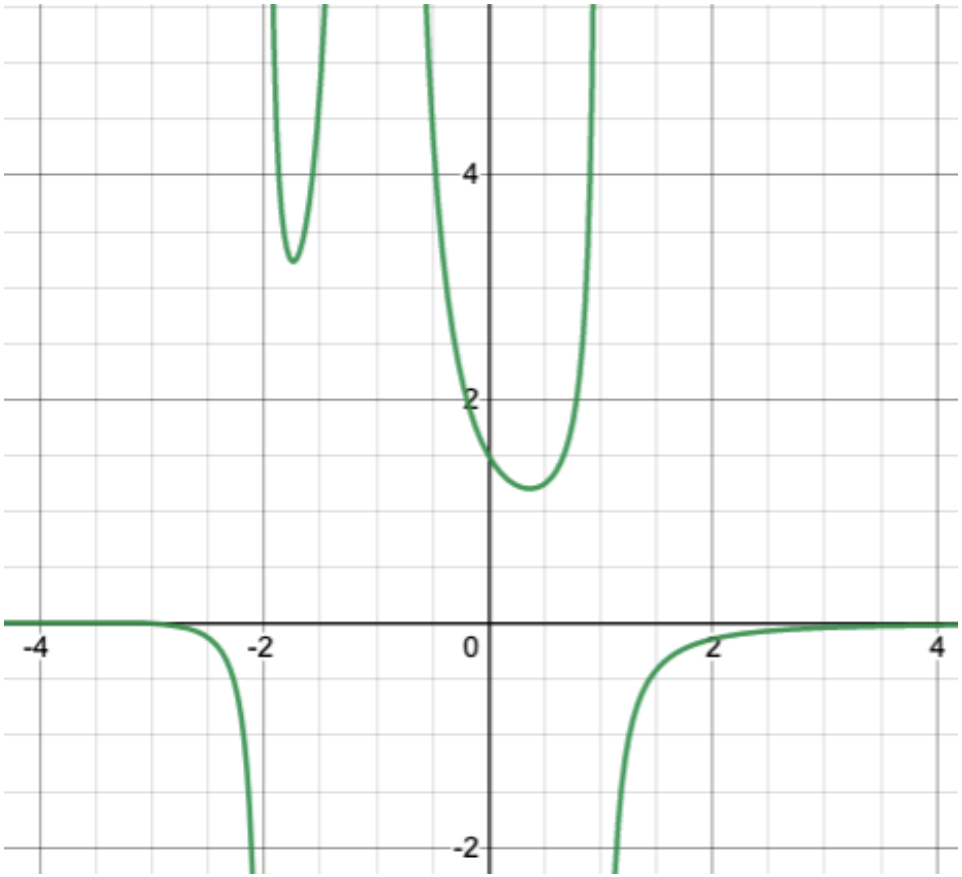
y = $-\frac{2}{9}$



7 Even More Graphing Rational Equations

Graphing

$$f(x) = \frac{-(x+3)}{(x+1)^2(x^2+x-2)}$$



Vertical asymptotes

$x = -1, -2, 1$

x-intercepts

$x = -3$

y-intercepts

$y = \frac{3}{2}$

Horizontal asymptotes

$y = 0$

End behavior

Left/right approach y=0

Behavior near vertical asymptotes

$1^+ = -\,,\ 1^- = +$

$-1^+ = +\,,\ -1^- = +$

$-2^+ = +\,,\ -2^- = -$

7 Even More Graphing Rational Equations

Finding Asymptotes

Vertical asymptotes occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator of equal or greater multiplicity.

X-intercepts occur at the zeros of the numerator which are not also zeros of the denominator.

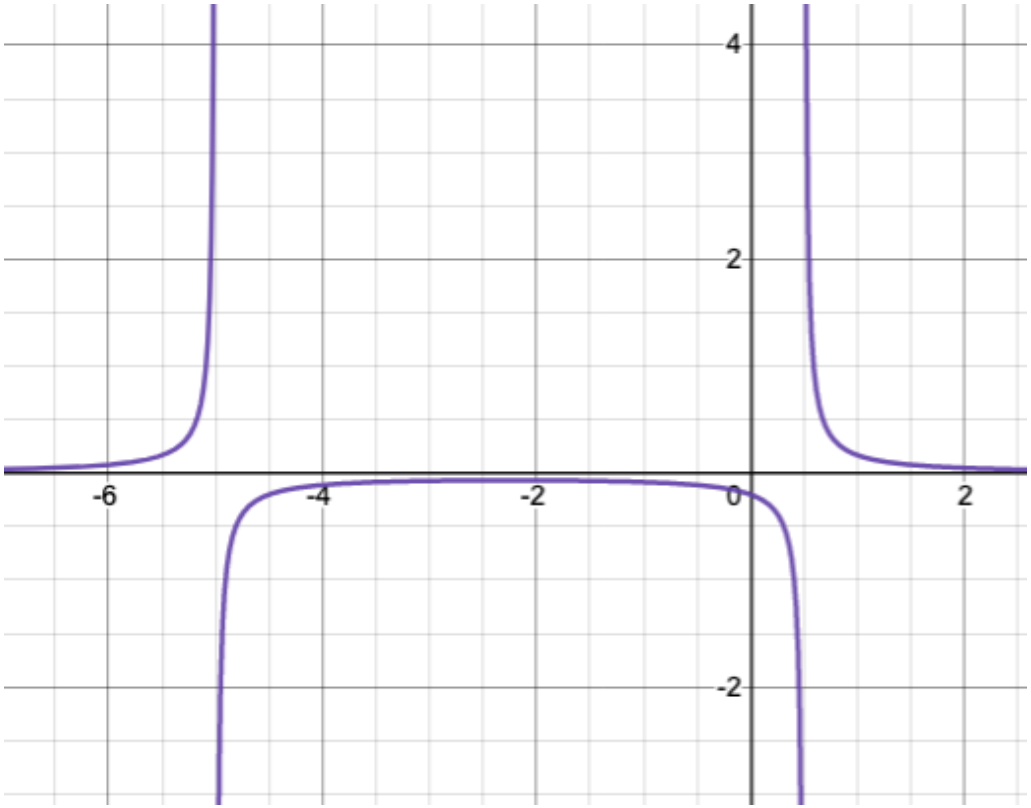
Y-intercept is the value of f(0) if defined.

	$f(x) = \frac{2x+1}{x+3}$	$f(x) = \frac{2x+1}{(2x+1)(x-3)}$	$f(x) = \frac{x+1}{(x+1)^2}$
Asymptotes:	$x = -3$	$x = 3$ Hole at x = $-\frac{1}{2}$	$x = -1$
x-intercepts:	$x = -\frac{1}{2}$	*none*	*none*
y-intercept:	$y = \frac{1}{3}$	$y = -\frac{1}{3}$	$y = 1$

7 Even More Graphing Rational Equations

Practice

$$f(x) = \frac{1}{2x^2 + 9x - 5}$$



Vertical asymptotes

$x = \frac{1}{2}, -5$

x-intercepts

none

y-intercepts

$y = -\frac{1}{5}$

Horizontal asymptotes

$y = 0$

End behavior

Left/right approach y=0

Behavior near vertical asymptotes

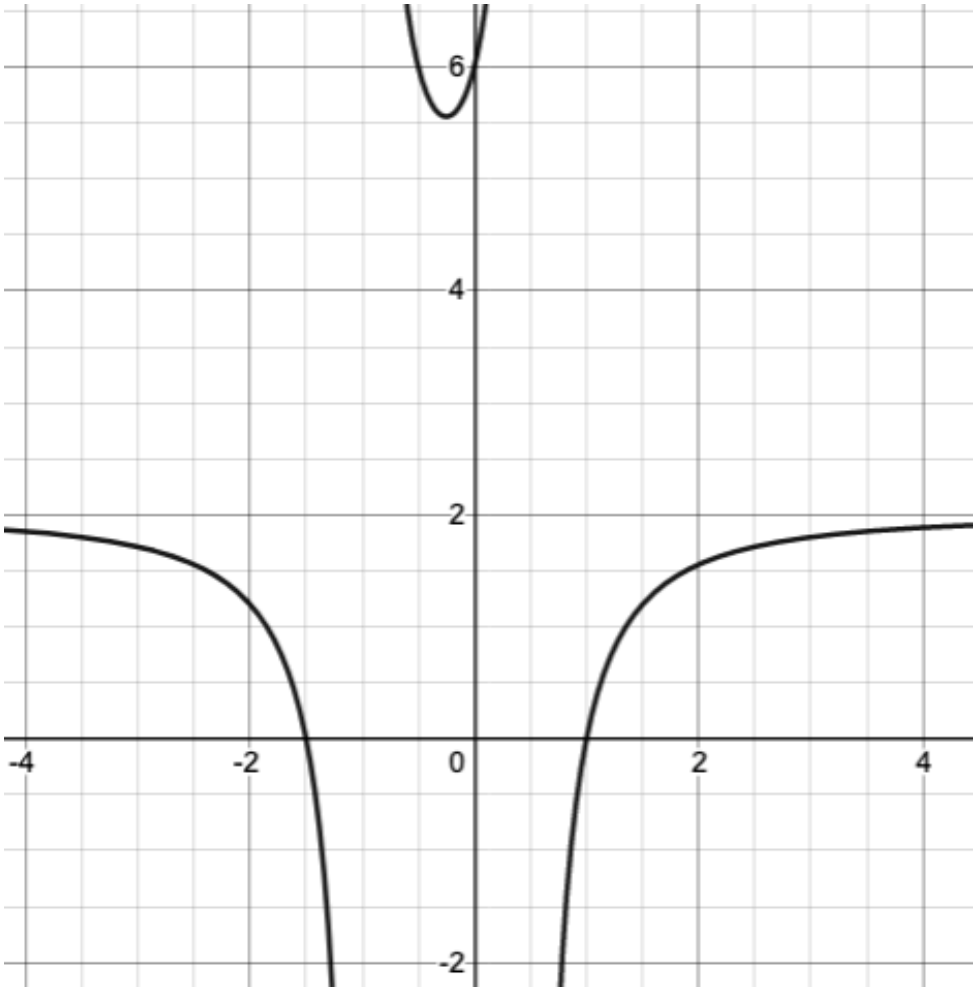
$\frac{1}{2}^{+} = \infty, \frac{1}{2}^{-} = -\infty$

$-5^{+} = -\infty, \ -5^{-} = \infty$

7 Even More Graphing Rational Equations

Practice

$$f(x) = \frac{-4}{2x^2 + x - 1} + 2$$



Vertical asymptotes

$x = \frac{1}{2}, -1$

x-intercepts

$x = -\frac{3}{2}, 1$

y-intercepts

$y = 6$

Horizontal asymptotes

$y = 2$

End behavior

Left/right approach y=2

Behavior near vertical asymptotes

$\frac{1}{2}^{+} = -\infty, \frac{1}{2}^{-} = \infty$

$-1^{+} = \infty, \ -1^{-} = -\infty$

Even More Solving Rational Equations

7 Even More Solving Rational Equations

$$\frac{3}{x^2-7x+10}+2=\frac{x-4}{x-5}$$

$$\frac{3}{(x-2)(x-5)}+\frac{2\cdot(x-2)(x-5)}{(x-2)(x-5)}=\frac{(x-4)\cdot(x-2)}{(x-5)\cdot(x-2)} \qquad x\neq 2,5$$

$$3+2\cdot(x^2-7x+10)=x^2-6x+8 \qquad x^2-8x+15=0$$

$$3+2x^2-14x+20=x^2-6x+8 \qquad (x-3)(x-5)=0 \qquad x=3,5$$

Practice

$$\left(\frac{x-3}{x+1}\right)^2=2\cdot\frac{x-3}{x+1}+3 \qquad x=1,-3$$

7 Even More Solving Rational Equations

$$\frac{3}{x^2-7x+10}+2=\frac{x-4}{x-5}$$

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$$3+2x^2-14x+20=x^2-6x+8 \qquad (x-3)(x-5)=0 \qquad x=3,5$$

Practice

$$\frac{5}{x^2+4x+4}=-4\cdot\frac{(x-3)}{x^2-x-6}+1 \qquad x=-3$$

7 Even More Solving Rational Equations

$$\frac{3x}{4} + 1 > \frac{x-1}{2}$$

$$\frac{3x}{4} + \frac{4}{4} > \frac{2(x-1)}{4}$$

$$3x + 4 > 2x - 2$$

$$x + 4 > -2$$

$$x > -6$$

$$\frac{(x+1)(x-2)}{(x+3)(x-1)} \geq 0 \ ?$$

$x < -3$ *or* $-1 \leq x < 1$ *or* $x \geq 2$

7 Even More Solving Rational Equations

$$\frac{3x}{4} + 1 > \frac{x-1}{2}$$

$$\frac{3x}{4} + \frac{4}{4} > \frac{2(x-1)}{4}$$

$$3x + 4 > 2x - 2$$

$$x + 4 > -2$$

$$x > -6$$

Practice

$$\frac{(x+2)(x-3)^2}{(x+4)(x-2)(x^2+5)} > 0$$

$-4 < x < -2 \text{ or } 2 < x < 3 \text{ or } x > 3$

hyperlink